**Fluid Statics Homework Problems**

**Swarthmore College Introductory Physics for the Life Sciences**

*Note: This problem set is designed to give you practice with pressure in gases and fluids, fluid statics, and surface tension.*

**Problem 1. Feeling Dizzy[[1]](#footnote-1)**

If you are lying down and stand up quickly, you can get dizzy or feel faint. This is because your circulatory system doesn’t have time to compensate for the change in height of your head, resulting in your blood pressure dropping at your head.

If your brain is 0.4 m higher than your heart when you are standing, what is the *difference between the blood pressure at your brain and that at your heart, as a percentage of the maximum[[2]](#footnote-2) blood pressure at the heart*? The density of blood plasma is about 1,025 kg/m3 and a typical maximum (systolic) pressure of the blood at the heart is 120 mg of Hg (= 16 kP = 1.6 x 104 N/m2).

**Problem 2. How Much Atmosphere is Up There?[[3]](#footnote-3)**

In our study of the pressure in fluids, we discovered that the pressure depended on depth because the fluid below is holding up the weight of all the fluid above it. When it comes to air, we are at the bottom of the atmosphere (near sea level). We can measure the pressure (~100 kPa) and the density (~1 kg/m3) of the air around us, but we don’t immediately know how “deep” we are in the atmosphere, i.e., we don’t immediately know how high above us the atmosphere goes. Let’s estimate it.

1. What is the weight of air being held up by a thin square of air just above the ground if the square has an area of 1 m2?
2. To get an idea of how high above us the atmosphere must go, assume that the air above us is of approximately uniform density, like water is.[[4]](#footnote-4) If that’s true, then what is the height *h* of the column of air you calculated in part a)?
3. Estimate the force with which the air in your dorm room pushes up on the ceiling of your room. You may approximate the density of the air in the room as a constant, as we did in part b).

**Problem 3. The Mystery of the Three Vases**[[5]](#footnote-5)

Water is poured to the same level in each of the three vessels shown. Each vessel has the same base area. Since the water is to the same depth in each vessel, each will have the same pressure at the bottom. Since the area and pressure is the same at the bottom of each vessel, each liquid should exert the same force on the base of the vessel. Yet, if the vessels are weighed, three different values are obtained. (The one in the center clearly holds less liquid than the one at the left, so it must weigh less.) How can you resolve this apparent contradiction?



**Problem 4. Under Pressure PhET Simulation[[6]](#footnote-6)**

The University of Colorado's PhET group has a very nice simulation that allows you to explore how pressure changes in a fluid due to the effects of gravity. It's called [Under Pressure](https://phet.colorado.edu/en/simulation/under-pressure), and if you follow the link you can either download the simulation or run it in your browser.

https://phet.colorado.edu/en/simulation/under-pressure



This is what it looks like when you run it in the browser, have turned on the ruler and the horizontal lines, and have moved the ruler so that it measures the distance below the top of the container (which is NOT necessarily the depth of the fluid). You can measure the pressure by using your mouse to grab the pressure meter and dragging it wherever you want. Note the the measurement is made at the little needle tip at the base of the meter. Be sure that your meter is set to the SI unit of pressures, Pascals (= 1 N/m2). The meter reports its results in kPa (kiloPascals = 1000 N/m2). For this problem use g = 9.80 N/kg.

Before you begin, turn on the grid and ruler and align the ruler as shown here to use as a coordinate system. We will report a reading on the ruler as a *y* coordinate.

1. First empty the container of liquid completely. Use the meter to measure the pressure at the top of the container and the bottom of the container. Now place the meter just above the top level of the container and fill the container with water. The measurement on your pressure meter should not have changed.
2. Write an equation that expresses how you think the pressure in the water changes as a function of *y* and calculate what pressure readings you expect at *y* = 0 m, *y* = 1 m, *y* = 2 m, and *y* = 3 m. Show how you are calculating your prediction and create a table like the one below comparing your prediction with the simulation's result. Do your predictions agree with the simulation? If you find a disagreement, explain it.

|  |  |  |
| --- | --- | --- |
| Depth (m) | Prediction (kPa) | Measured (kPa) |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

1. Now make a prediction for what you think the pressures would be if you turned off the atmosphere. Show how your calculations change from the previous case and test your prediction against the simulation, making another table and explaining any discrepancies you find.

1. Now switch to the conical vessels -- the ones chosen by this button -- and fill them up to the top.  
      
   Before making a measurement, make a prediction how the pressure in the two vessels will compare in the center of each cone, 1m down and just above the bottom. Which do you expect to be bigger and why? Now make the measurement and give the result.

**Problem 5. Fish Buoyancy[[7]](#footnote-7)**

To stay at a constant depth, an aquatic organism needs to be *neutrally buoyant*, meaning that the buoyant force on it and the force of gravity on it have equal strengths.  To achieve this, the animal must have an average density that is equal to that of water, so that the mass of its body is equal to the mass of the volume of water being displaced.

One of the complications to maintaining neutral buoyancy underwater is the fact that pressure changes with depth.  Because air volume is inversely proportional to pressure, as we will see later in the course (and as you may already know if you have studied the ideal gas law), the volume of air in a fish’s swim bladder or a mammal’s lungs decreases with depth, changing the animal’s overall volume and hence its overall density. The following problem explores some of the consequences of this.

1. Consider a freshwater fish swimming in water of density 1,000 kg/m3.  Assume that the average density of its tissues is 1,070 kg/m3, and its swim bladder makes up 7% of its total volume at the surface.  Calculate the average density of the fish at the surface.
2. What happens to the fish’s average density as it descends to a point that is 20 m below the surface?  What would happen to the fish if it did nothing?
3. What volume of air would have to be added to the swim bladder for the fish to remain neutrally buoyant at a depth of 20 m?

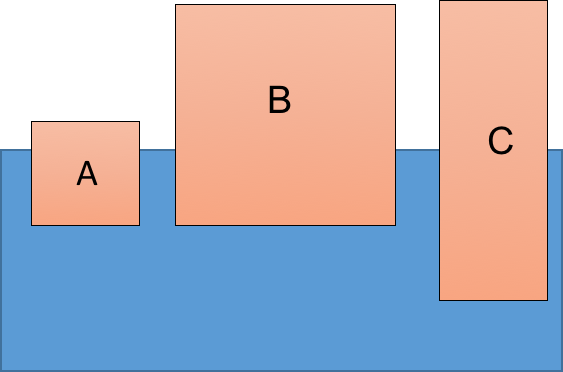
**Problem 6. Pixar Science[[8]](#footnote-8)**



In the Disney-Pixar movie *Up*, a disgruntled and bored old man attaches a large number of helium balloons to his house and floats away to have some adventures. However, it seems unlikely that the materials the roof is made of are strong enough that the house could be lifted from one point on the roof without breaking apart. So let’s consider a somewhat more plausible situation:

Estimate how many helium balloons of the size shown in the figure at the right[[9]](#footnote-9) would be needed to lift the kitten. You can take the density of air to be ~ 1 kg/m3 and the density of helium to be about 1/7 that of air when both are at room temperature and pressure. (*Hint: This is one case in which the buoyant force on an object in air does matter!*)

**Problem 7. Floating Blocks[[10]](#footnote-10)**

The figure shows three blocks floating in salt water, which has a density that is 1.1 times that of fresh water.[[11]](#footnote-11)

Block *a* is floating with 75% of its volume submerged, block *b* is floating with 33% of its volume submerged, and block *c* is floating with 50% of its volume submerged.

1. Find the density of block *b* in terms of the density of the salt water it is floating in, . Show your work (including a free body diagram) and make your logic clear.
2. Now some fresh water is added to the container to decrease the overall density of the salt water to 1.05 times that of . Will the blocks continue to float with the same fractions of their volume submerged, change the level at which they float, or sink? Explain briefly. If they change the level at which they float, specify clearly whether more or less of the blocks will be submerged and why.

**Problem 8. A Concrete Canoe[[12]](#footnote-12)**

Each year, schools design, build, and race full-size canoes made entirely of reinforced concrete. These canoes are judged in regional competitions based on design and construction quality, written presentation of work performed, and racing performance. The “best” canoe from each region then moves on to a professionally sponsored national competition. How can you make a canoe out of concrete that floats?

**Problem 9. The Water Strider**

The water strider uses surface tension to support itself as it walks on the water. As shown in the photo, its six feet are elongated so that they have a long perimeter on which they contact the water, in the same manner that a needle can be floated on water.

****Modeling the needle or the elongated foot of the water strider as a long cylinder with length much greater than its diameter, the strength of the upward surface tension force on it can be estimated as[[13]](#footnote-13) . The value of the surface tension for water is 0.073 N/m.

Estimate the maximum mass of a water strider in grams (or kilograms if you prefer). Include a free body diagram for the water strider when all of its feet are on the water as part of your explanation. Use the photo to help you estimate the length of each of the water strider’s feet.

**Problem 10. Lung Surface Area**

We mentioned in class that lungs are made up of many tiny alveoli, which increases the overall surface area of the lungs. This is important because the rate of exchange of oxygen and carbon dioxide depends on the size of this surface area. Estimate the following:

1. the surface area that lungs would have if they consisted of two bags occupying the upper half of a human torso
2. the actual surface area provided by the alveoli (according to Wikipedia, [[14]](#footnote-14) the lungs of an adult male have about 300 million alveoli ranging in diameter from 75 µm to 300 µm)
3. the ratio of your results for (a) and (b)

In your solution, make clear the model that you use to calculate these surface areas and any additional assumptions you make beyond the information provided in the problem.[[15]](#footnote-15)

**Problem 11. The Law of Laplace**

In class we discussed the relationship between surface tension, radius, and pressure difference for a spherical bubble written in the form:

This relationship is sometimes called the Law of Laplace.

Although for water the surface tension is a constant, for lung surfactant the surface tension depends on its density on the inside surface of the alveoli (tiny spherical parts of the lung tissue). This density varies repeatedly while a person breathes, because the alveoli get larger when inhaling and smaller when exhaling.

1. If the same number of molecules of lung surfactant coat the inside of an alveolus at all times, does the *area density* (number of molecules per area) of surfactant increase or decrease when the alveolus gets larger? (No explanation required)
2. When a person inhales, the diaphragm forces the lungs to expand (increase their volume) by pulling the bottom of the lungs down. When this happens, for air to be drawn in, the radii of all the alveoli also need to expand. However, if air comes in, the pressure also needs to increase. So when inhaling, both and *r* should increase. Does require that the surface tension also increase, decrease, or remain the same? Show your work and make sure that your logic is clear.
3. What do your results from parts a) and b) suggest about how the surface tension of lung surfactant should depend on its area density?[[16]](#footnote-16)

[Also assigned problems from Knight, Physics for Scientists and Engineers, 4th edition: Chapter 15, Problems 18 and 35.]

1. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-1)
2. Maximum here refers to maximum during the cardiac cycle; the pressure goes up and down as the heart beats. [↑](#footnote-ref-2)
3. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-3)
4. This isn’t true, but it’s good enough for this back-of-the-envelope calculation. [↑](#footnote-ref-4)
5. From A. Arons, *A Guide to Introductory Physics Teaching* (John Wiley & Sons, Inc. 1990). [↑](#footnote-ref-5)
6. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-6)
7. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-7)
8. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-8)
9. From [http://icanhascheezburger.com](http://icanhascheezburger.com/) [↑](#footnote-ref-9)
10. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-10)
11. Sea water is only about 3% denser than fresh water; the density depends on the amount of salt. [↑](#footnote-ref-11)
12. Problem from NEXUS/Physics, with minor adaptations [↑](#footnote-ref-12)
13. The factor of 2 comes because the surface tension force acts on each of the long sides of the cylinder. Thus the total surface tension force equals the perimeter of the needle where it contacts the water multiplied by the surface tension; as the needle’s length is much greater than its diameter, the perimeter is approximately 2*L*. [↑](#footnote-ref-13)
14. Accessed online at <https://en.wikipedia.org/wiki/Gas_exchange> on October 21, 2015. [↑](#footnote-ref-14)
15. For example: for part (a), what shape do you assume for the bags? What values do you assume for the torso dimensions, and how did you arrive at those values? Possible ways to arrive at those values could include: measuring yourself or a friend with a tape measure, estimating based on your knowledge of an average person’s height and assuming that the torso is a certain fraction of that height, knowing typical dimensions based on buying clothes, etc. … [↑](#footnote-ref-15)
16. You should find that the surface tension increases as the area density decreases. For those of you who are interested in chemistry: we said that surface tension comes from the force required to distort or stretch a surface due to the interaction of the molecules of the surface with the air above. Think about whether you can come up with a molecular explanation for why the relationship you found in part c) makes sense. [↑](#footnote-ref-16)